On isometries satisfying deformed commutation relations

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We consider certain perturbation of family of pairwise orthogonal isometries. Namely, we study properties and representation theory of C^*-algebra $\mathcal{E}_{1,n}^q$ generated by isometries t, s_j, $j=\overline{1,n}$, subject to the relations

$$s_i^* s_j = 0,\ i \neq j, \quad t^* s_j = q s_j t^*.$$

In recent paper [1] was studied the C^*-algebra $\mathcal{E}_{n,m}^q$ with $n, m \geq 2$, generated by families $\{t_j\}_{j=1}^m$ and $\{s_i\}_{i=1}^n$. In particular, it was shown that for $|q| < 1$ one has $\mathcal{E}_{n,m}^q \simeq \mathcal{E}_{n,m}^0$ and for $|q| = 1$ the C^*-isomoprphism class of quotient of $\mathcal{E}_{n,m}^q$ by the unique largest ideal is independent of q and isomorphic to the tensor product of Cuntz algebras $\mathcal{O}_n \otimes \mathcal{O}_m$.

We show that the result for $|q| < 1$ remains true for $\mathcal{E}_{1,n}^q$.

Theorem 1. *For any $q \in \mathbb{C}$, $|q| < 1$, one has an isomorphism $\mathcal{E}_{1,n}^q \simeq \mathcal{E}_{1,n}^0$.*

Notice that the proof contains an explicit construction of the required isomorphism, which is similar to the one given in [1].

For the case $|q| = 1$ we obtain the following facts.

Definition 2. The Fock representation, π_F^q, of $\mathcal{E}_{1,n}^q$, is the unique up to unitary equivalence irreducible $*$-representation having the vacuum vector Ω, $||\Omega|| = 1$, such that

$$\pi_F^q(s_j^*)\Omega = 0, \quad \pi_F^q(t^*)\Omega = 0, \quad j = \overline{1,n}.$$

Theorem 3. *The Fock representation of $\mathcal{E}_{1,n}^q$ exists and is faithful.*

Theorem 4. *The C^*-algebra $\mathcal{E}_{1,n}^q$ is nuclear.*

Also we prove an analog of Wold decomposition Theorem for such family of isometries, and study irreducible representations corresponding to each of its components.

References

[1] A. Kuzmin, V. Ostrovskyi, D. Proskurin and R. Yakymiv. On q-tensor product of Cuntz algebras, preprint (2019), https://arxiv.org/abs/1812.08530